Name: Akmal Raza

Name of the supervisor: Dr. Arshad Khan

Name of the co-supervisor: Prof. Khalil Ahmad

Title of the thesis: A Study on Applications of Wavelets

The wide range of applications of wavelets in modern world has drawn attention of most researchers. The wavelet methods have shown its importance for solving ordinary, partial and fractional deferential equations. The usage of wavelet methods in signal and image processing has found a considerable interest of many researchers. Present work was divided into seven chapters.

The study is highly motivated from the paper in which author discussed wavelet transforms versus Fourier transforms. In fact, we try to answer three questions arising in wavelets theory before study of wavelets. Why, what and how to use wavelets? For motivation, we started from Weierstrass Approximation theorem, Taylor's theorem, Fourier transform and then wavelets. Also, we discuss what after the wavelets (wavelet packets, framelets, curvelets, shearlets and linear canonical wavelet transform). Also, we solved linear and nonlinear neutral delay differential equations using Haar wavelet series method. We apply Haar wavelet and obtain its integration for neutral delay differential equations with respect to the collocation points to obtain the numerical solution of the problems. Further, we solved linear and nonlinear neutral delay differential equations using Haar wavelet series method. Further, we solve the linear and nonlinear second order singularly perturbed differential difference equations and singularly perturbed convection delayed dominated diffusion equations, arising in various modelling of chemical processes. First, we transform delay term by using Taylor expansion and then apply Haar wavelet method. To show the robustness, accuracy and efficiency of the method, three problems of second order singularly perturbed differential difference equations and three problems of convection delayed dominated diffusion equations have been solved. Further, numerical solution of system of first order linear partially singularly perturbed initial value problem on piecewise uniform Shishkin mesh and ρ-mesh. Further, we apply same technique for solving system of second order linear partially singularly perturbed boundary value problems on piecewise uniform Shishkin mesh andq-mesh. We develop a new scheme based on finite-difference and Haar wavelet for second order diffusion equation and third order dispersive equation.

Further, we have carried out the stability of the Haar wavelet. We solved four problems consisting of linear diffusion equation and dispersive homogeneous and non homogeneous equations to validate the developed scheme. We solved fourth order parabolic partial differential equation. We approximate space derivative by Haar wavelet and time derivative by finite-difference to find the solution of fourth order parabolic partial differential equation.

In last we suggest a method based on anisotropic equation and wavelet transform to denoise the noisy images. We solved the anisotropic equation or Perona and Malik model by taking noisy images as initial solution using finite difference method and then we applied wavelet de-noising scheme on thesolution of anisotropic equation and obtain denoise images. Further, we calculated and tabulated the PSNR values and compared with existing methods.

In future, wavelets can be applied in cryptography, coding theory and fractional differential equations. Wavelet, can solve any type of differential equations such as ordinary, partial, singularly perturbed BVP, delay, neutral delay, delay in PDE, neutral delay in PDE and higher dimensional PDE with delay and neutral delay. Also, the higher order fractional ODE and PDE, fractional order ODE, PDE with delay and neutral delay arising in various mathematical modelling of real life problems such as auction price, stock market, human behaviour, neural network, image processing, neuronal variability, micro scale heat transfer, semiconductor mass device models. The solution of differential equations on sphere, ellipsoid or three dimensional surfaces are on high demand and the wavelet on manifold produces approximate solution for this kind of problems with fast computation. Also, the solution of differential equations on irregular domain are hot topics for present and future so the wavelet can be applied for solving these kind of problems. Further, for those researchers who want to work on pure analysis can work on the basis constructions of Sobolev space, Schwartz space and Besov space. The spaces for the solution of fractional order differential equations are high in demand and wavelets can help for construction of basis set for these kinds of spaces.