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**Title:** A Study of Ideals in Algebraic System

## ABSTRACT

In Abstract Algebra, the concept of ideals, quasi-ideals, bi-ideals and  $(m, n)$ -ideals are well known. A lot of work has been done on these ideals by many algebraists in different algebraic structures. However, the very fundamental results of generalized ideals, covered lateral ideals and generalised quasi-ideals in ternary structures, and soft hyperideals in different hyperstructures etc. remained untouched. Therefore a need was felt to study and generalize these ideals in different algebraic structures. The present thesis comprises eight Chapters consisting of various sections.

**Chapter 1.** In Chapter one, some basic definitions and results which are needed to prove the results in the subsequent Chapters have been stated. Most of these results and definitions are available in standard references on the subject. It serves the purpose to acquaint the reader with the terminology and basic facts often used and also to make thesis as much self-contained as possible.

**Chapter 2.** In Chapter two, we characterize the relationship between minimal  $m$ -right, minimal  $(p, q)$ -lateral, minimal  $n$ -left ideal and  $m$ -right simple,  $(p, q)$ -lateral simple,  $n$ -left simple ternary semigroups. Further, we introduce the notion of covered lateral ideals in ternary semigroups. We prove that if a ternary semigroup  $T$  contains two different lateral ideals  $L_1$  and  $L_2$  such that  $L_1 \cup L_2 = T$ , then none of the lateral ideals  $L_1, L_2$  are covered lateral ideals. We also give some examples in support of our discussion. The content of this Chapter is published in Quasigroups and Related Systems(Moldova).

**Chapter 3.** Chapter three deals with generalised quasi-ideals in ordered ternary semigroups. We also define ordered  $m$ -right ideals, ordered  $(p, q)$ -lateral ideals and ordered  $n$ -left ideals in ordered ternary semigroups and studied the relations among them. Some intersection properties of ordered  $(m, (p, q), n)$ -quasi ideals are examined. We characterize these notions in terms of minimal ordered  $(m, (p, q), n)$ -quasi-ideals in ordered ternary semigroups. Moreover, we define  $m$ -right simple,  $(p, q)$ -lateral simple,  $n$ -left simple, and  $(m, (p, q), n)$ -quasi simple ordered ternary semigroups and some of their properties are studied. We also give some examples in support of our discussion. The content of this Chapter is published in Kyungpook Mathematical Journal(Korea).

**Chapter 4.** In Chapter four, we define soft intersection (briefly, S.I.) product of soft sets in  $\Gamma$ -semihypergroup and prove that this product satisfies associative law. Further, we introduce the notions of S.I. left, S.I. right, S.I. bi, S.I. interior, S.I. quasi  $\Gamma$ -hyperideals of  $\Gamma$ -semihypergroups over an initial universe  $V$  and define these S.I.  $\Gamma$ -hyperideals with soft intersection(S.I.) product. Also, we introduce the notion of  $(\xi, \zeta)$ -soft  $\Gamma$ -hyperideals and  $(\xi, \zeta)$ -soft interior  $\Gamma$ -hyperideals of  $\Gamma$ -semihypergroups by a new approach called soft intersection(S.I.). It is proved that in regular  $\Gamma$ -semihypergroups  $(\xi, \zeta)$ -soft  $\Gamma$ -hyperideals and  $(\xi, \zeta)$ -soft interior  $\Gamma$ -hyperideals coincide. Further, we introduce  $(\xi, \zeta)$ -soft simple  $\Gamma$ -semihypergroup and characterize the simple  $\Gamma$ -semihypergroups in terms of  $(\xi, \zeta)$ -soft  $\Gamma$ -hyperideals and  $(\xi, \zeta)$ -soft interior  $\Gamma$ -hyperideals. We also give some examples in support of our discussion. The content of this Chapter is published in Honam Mathematical Journal(Korea).

**Chapter 5.** Chapter five is devoted to the study of soft hyperideals in LA-semihypergroups through new approach and also we have investigated some interesting results. We characterize regular and intra-regular LA-semihypergroups in terms of soft hyperideals. Moreover, we also give an example in support of our discussion. The content of this Chapter is accepted in AISC Book Series of Springer(Singapore).

**Chapter 6.** In Chapter six, we introduce the notion of soft interior-hyperideals. Further, we give several basic properties of these notions and also discuss some characterizations in terms of soft interior hyperideals. We also give an example in support of our discussion. The content of this Chapter is accepted in Kragujevac Journal of Mathematics(Serbia).

**Chapter 7.** In Chapter seven, we introduce soft hyperideals in ordered LA-semihypergroups through new approach and investigated their properties. Then, we define right regular ordered LA-semihypergroup and characterize right regular ordered LA-semihypergroups using soft hyperideals. We also give an example in support of our discussion. The content of this Chapter is accepted in AISC Book Series of Springer(Singapore).

**Chapter 8.** In Chapter eight, we have collected the findings and conclusions of the Chapters of the thesis. Also, we have discussed the future work that can be done on these structures. In the end, a comprehensive bibliography with the author's name in alphabetical order is given enlisting books and papers which have been referred to in the thesis.

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