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Title: **A Study of Certain Submanifolds and Applications**

ABSTRACT

The study of "*Geometry of submanifolds of a differentiable manifold*" is one of the most captivating topics of modern differential geometry. This can be generalized as the study of curves and surfaces in 3-dimensional Euclidean space to arbitrary dimensions and co-dimensions and to arbitrary ambient spaces. The contributions of this study would be interesting to researchers in differential geometry and other related fields. The main objective of this thesis is to study the numerous geometric structures on any submanifold of a Riemannian manifold. The research presented in this thesis has produced some results and immediate geometric applications also. The thesis is divided into six chapters and each chapter is divided into several subsections. A short overview of this thesis is as follows:

Chapter 1 is introductory. In this chapter, we review the theory of Riemannian manifolds and submanifolds. This chapter is intended to collect all the theory that will be essential for the following chapters and to give the notation for the rest of the thesis.

Chapter 2 is divided into four sections. This chapter deals with much known formula, that is, Simons' type formula. Firstly, we prepare a preliminary lemma and quote some important lemmas for later use. Next, we construct Simons' type formula for a minimal Kaehlerian slant submanifold of a complex space form. Secondly, we classify a minimal Kaehlerian slant submanifold of a complex space form by using derived formula. We also provide some immediate consequences of the main theorem. Thirdly, we deal with different semi-parallel submanifolds in a Kahler manifold with certian conditions. Finally, we construct some examples of slant, semi-slant, hemi-slant, bi-slant and CR submanifolds in almost complex manifolds.

This chapter has been published in **Kyungpook Mathematical Journal**, 2018 (**SCOPUS and ESCI**).

Chapter 3 is divided into four sections. This chapter is fully devoted to the optimal $\boldsymbol{\delta}$-Casorati inequalities for submanifolds in an odd dimensional Riemannian manifold. Firstly, we study $\boldsymbol{\delta}$-Casorati curvatures for submanifolds in an odd dimensional Riemannian manifold and mention some notations for later use. Secondly, we construct a family of inequalities that relate the normalized scalar curvature with the normalized $\boldsymbol{\delta}$-Casorati curvature for bi-slant submanifolds of generalized Sasakian space forms. We also show that the equality at all points characterizes the invariantly quasi-umbilical submanifolds in both cases. Thirdly, we establish two sharp inequalities, involving the generalized normalized $\boldsymbol{\delta}$-Casorati curvatures and the generalized normalized scalar curvature of any submanifold in generalized Sasakian space forms endowed with semi-symmetric metric connection by using T. Oprea's technique. We also examine that the equality holds if and only if the submanifold is invariantly quasi-umbilical in both inequalities. Finally, we discuss these inequalities for different submanifolds such as invariant, anti-invariant, CR, slant, semi-slant, pseudo-slant submanifolds in the same ambient space. We see a glimpse of these inequalities in different structures such as Sasakian space form, Kenmotsu space form and cosymplectic space form. Finally, we develop these obtained inequalities for invariant, anti-invariant, CR, slant, semi-slant,pseudo-slant and bi-slant submanifolds in the same ambient space form endowed with SSMC.

Results of this chapter have been published in **Acta Mathematica Universitatis Comenianae**, 2018 (**SCOPUS and ESCI**) *and* **International Electronic Journal of Geometry**, 2018 (**ESCI**).

Chapter 4 is divided into four sections. This chapter deals with the study of the warped product bi-slant submanifolds of nearly trans-Sasakian manifolds. Firstly, we prove some results and prepare some preliminary lemmas, which shall be required in proving the results. Secondly, we prove that the squared norm of the second fundamental form for bi-slant submanifolds with any co-dimension of nearly trans-Sasakian manifolds is bounded below by the gradient of a warping function and also find the conditions on which the equality holds. Thirdly, we see the triviality of warped product bi-slant submanifolds with certain conditions in the same ambient manifold and also give some immediate applications. Finally, some related examples are provided.

This chapter has been published in **Journal of Inequalities and Applications**, 2018 (**SCI**).

Chapter 5 is divided into six sections. This chapter gives an introduction to the statistical manifolds and their submanifolds. Firstly, we obtain some results by using fundamental properties of totally real statistical submanifolds immersed into holomorphic statistical manifolds. Secondly, we study and provide several results on CR-statistical submanifolds in the same ambient manifold. Thirdly, we prove some results on totally umbilical CR-statistical submanifolds of holomorphic statistical manifolds of constant holomorphic sectional curvature. Fourthly, we give necessary and sufficient condition for a CR-statistical submanifold to be a CR-product in holomorphic statistical manifolds and obtain its some immediate consequences. Finally, under certain geometric conditions, we prove a result on proper CR-statistical submanifolds in the same ambient manifold. This chapter finishes with some related examples.

Results of this chapter have been published in **GSI'17**, Springer (**SCOPUS**) *and* **Filomat**, 2018 (**SCI**).

Chapter 6 is divided into five sections. This chapter is fully devoted to some geometric inequalities for statistical submanifolds with any co-dimension and statistical real hypersurfaces of holomorphic statistical manifolds of constant holomorphic sectional curvature. Firstly, we study Casorati curvatures for statistical real hypersurfaces. Secondly, we prove that the normalized scalar curvature for any statistical real hypersurface of a holomorphic statistical manifold of constant holomorphic sectional curvature is bounded above by the generalized normalized Casorati curvatures. We also discuss the characterisation of equality cases. Thirdly, we construct the generalized Wintgen inequality for Lagrangian statistical submanifolds in the same ambient by using different approach. Fourthly, we obtain some general inequalities for statistical real hypersurfaces and totally real statistical submanifolds in the same ambient. Finally, we discuss some geometric applications of the derived inequalities.

Results of this chapter have been published in **Filomat**, 2018 (**SCI**).

Chapter 7 is divided into two sections. This chapter mainly focuses to summarize this thesis, discuss its findings and also outline the directions for future research work.