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Title: **Numerical Solution of the System of Nonlinear Singular Two Point Boundary Value Problems on a Variable Mesh**

Abstract

Ordinary differential equations are used to represent various phenomena in the field of physics, engineering, biology and other areas of technology. More specifically, a wide variety of natural phenomena are modelled by second-order linear and nonlinear initial and boundary value problems (BVPs). There has been a considerable amount of work for solving second order problems. To name a few, Wazwaz [Appl. Math. Comput., 128(2002), 45-57] applied the Adomian decomposition method to solve singular second-order initial value problems, Ramos [Appl. Math. Comput, 161(2005), 525-542] proposed linearization techniques for solving second-order singular initial value problems of ordinary differential equations. Bougoffa [Appl. Math. Lett., 22(8)(2009),1248-1251] solved initial value problems of a class of second-order linear differential equations by writing the equations in the general operator form and derived an inverse differential operator for the same. Boutayeb [Int. J. Comput. Math., 45(3)(1992),207-223] developed family of finite difference methods to solve boundary value problems. Russell [Numer. Math., 19(1972),1-28.] used collocation method for solving second order BVPs. Khaled et. al. [Appl. Math. Model., 31(2)(2007),292-301] used Adomian decomposition method to develop a fast and accurate algorithm for solving a special second-order ordinary initial value problems. However, many classical numerical methods used to solve second-order initial value problems can't be applied to second-order boundary value problems. In most of the cases, the analytical solution of the problem cannot be obtained although the existence and uniqueness of the solution is easy to establish. Hence, numerical methods for solving boundary value problems (i.e., scalar or vector) in scientific and engineering problems are not only feasible but also very enticing. The availability of rich literature in case of second order problems made it desirable to use them to solve system of second order as well as higher order nonlinear singular two point boundary value problems. The present work is devoted to the study of finite difference method, cubic spline and non-polynomial spline methods based on variable meshes. The methods are discussed for uniform mesh too. We have developed the finite difference method of different order based on variable mesh to obtain the numerical solution of system of nonlinear singular boundary value problems. We also provide the computational order of convergence of fourth order method based on uniform mesh in each chapter to confirm order of convergence of the third order method. The research work presented in this thesis consists of seven chapters. The chapter-wise summary is given herewith:

Chapter 1 contains a brief introduction to boundary value problems (BVPs) with its existence and uniqueness theorem. We also introduce singular boundary value problems, nonlinear singular boundary value problems, system of

boundary value problems. A survey of finite difference methods, cubic spline functions with an algorithm to solve block matrices developed due to discretization of BVPs i.e., Block Gauss Elimination Method and Block Newton's Method are also provided.

In Chapter 2, we develop two generalized finite difference scheme based on variable mesh for the numerical solution of system of second-order nonlinear singular two point boundary value problems. The proposed variable mesh method is second and third-order accurate. Convergence analysis of the third order method is discussed. Numerical examples are given to illustrate the efficiency of our methods.

In Chapter 3, two variable mesh finite difference scheme based on off-step points are developed to solve system of second order nonlinear singular two-point boundary value problems. The proposed variable mesh method is second and third-order accurate. Convergence analysis for fourth order uniform mesh method is provided. Numerical results are provided to affirm the primacy of our methods.

In Chapter 4, we developed cubic spline scheme based on variable mesh together with another variable mesh scheme for obtaining smooth approximations to the numerical solution of system of two point boundary value problems. We also define higher order approximations of first order derivatives using cubic spline and use them in the second scheme for better results. It is shown that the present methods give approximations better than other existing methods. Seven numerical examples are given to demonstrate the advantages of our methods.

In Chapter 5, non-polynomial spline scheme based on variable mesh, which reduces to cubic polynomial splines, have been used to develop two variable mesh numerical methods for the solution of system of two-point nonlinear singular boundary value problems. Convergence analysis of the third order method using fourth order boundary value problem is discussed through standard procedure. Also application of the methods is shown on fourth, sixth order BVP and on a coupled boundary value problem. It is shown that the present method gives better approximations. Eight numerical examples are given to illustrate the practical usefulness of our approach.

In Chapter 6, two variable mesh non-polynomial spline methods based on off-step points are developed for the solution of system of two point nonlinear singular boundary value problems. Vector convergence analysis using fourth order boundary value problem is discussed .Also application of the methods are shown on fourth and sixth order BVPs. It is shown that the present method gives better approximations. Six numerical examples are given to illustrate the utility of the proposed methods.

In Chapter 7, we developed finite difference scheme based on variable mesh for solving a system of nonlinear boundary value problem with mixed boundary conditions. The fourth order BVP is first decomposed into system of boundary value problems and then the system is discretized. Separate third order scheme is also developed at the end points. Comparisons are made with fourth order uniform mesh method.

At last, a comprehensive bibliography has been provided.