Name of Ph.D. Scholar : **Mohd. Shaid Baboo**
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Title of the Ph.D. : **Exact Solutions of Outstanding Problems and Novel Proofs Through Hypergeometric Approach**

Abstract

Special functions and their applications are now awe-inspiring in their scope, variety and depth. Not only in their rapid growth in pure Mathematics and its applications to the traditional fields of Physics, Engineering and Statistics but in new fields of applications like Behavioral Science, Optimization, Biology, Environmental Science and Economics, etc. they are emerging. The aim of the first chapter is to introduce several classes of special functions, which occur rather more frequently in the study of summations and transformations needed for the presentation of subsequent chapters.

Some significant hypergeometric summation theorems with suitable convergence conditions are obtained in second chapter, which are analogous to Kummer's summation theorem ${}_2F_1(-1)$ recorded by Prudnikov *et al.* and derived by Choi, Kim *et al.*, Rakha-Rathie and Rathie-Kim. By means of these summation theorems we also find the Laplace transforms of Kummer's confluent hypergeometric function ${}_1F_1$ in closed form.

Some significant hypergeometric summation theorems with suitable convergence conditions, are obtained in third chapter and fourth chapter; analogous to summation theorems for Gauss function ${}_2F_1(\frac{1}{2})$ presented by Brychkov, Prudnikov *et al.* and derived by Fox, Rakha-Rathie. By means of these summation theorems we also find the Laplace transforms of Kummer's confluent hypergeometric function ${}_1F_1$ in closed form.

In fifth chapter and sixth chapter, we find summation theorems for generalized hypergeometric functions ${}_pF_{q+1}[(\alpha_p); g\pm m, (\beta_q);\ z]$ and ${}_{p+1}F_q[a\pm m, (\alpha_p); (\beta_q);\ z]$ respectively by means of Mellin-Barnes type contour integral representation. Some closed forms (believed to be new) for Clausen hypergeometric functions ${}_3F_2[-1]$, etc. and Gauss hypergeometric functions ${}_2F_1[-1]$ etc. with suitable convergence conditions, are also obtained.

In seventh chapter, we find the power series representations and single, Kampé de

Fériet's double, Srivastava's triple, Saigo's quadruple and Srivastava-Daoust multiple hypergeometric forms of positive integral powers of arcsine functions. Using some definite integrals involving arcsine functions, we obtain some transformations for terminating and truncated hypergeometric series ${}_4F_3(1)$, some summation theorems and transformations for Kampé de Fériet double hypergeometric functions, Srivastava's triple hypergeometric functions with positive unit arguments, in terms of Riemann's zeta-functions, Catalan's constant, Polylogarithm functions and Polygamma functions. We also establish some mixed relations involving single, double and triple hypergeometric functions.

In eight chapter, we find the power series representations and single, Kampé de Fériet's double, Srivastava's triple, Saigo's quadruple and Srivastava-Daoust multiple hypergeometric forms of positive integral powers of arctan functions. Using some definite integrals involving arctan functions, we obtain some transformations for truncated hypergeometric series ${}_3F_2(\pm 1)$, some summation theorems for Kampé de Fériet double hypergeometric functions, Srivastava's triple hypergeometric functions and quadruple hypergeometric functions of Saigo with arguments (-1), in terms of Riemann's zeta-functions, Catalan's constant, Polylogarithm functions and Polygamma functions. We also establish some mixed relations involving single, double triple and quadruple hypergeometric functions.

In ninth chapter, we find the power series representations and single, Kampé de Fériet's double, Srivastava's triple, Saigo's quadruple and Srivastava-Daoust multiple hypergeometric forms of various positive integral powers of $\ell n[1 \pm z]$ functions. Using some definite integrals involving $\ell n[1 \pm z]$ functions, we obtain some reductions formulas for truncated hypergeometric series ${}_3F_2(\pm 1)$ in terms of truncated hypergeometric series ${}_2F_1(\pm 1)$, some summation theorems for Kampé de Fériet double hypergeometric function, Srivastava's triple hypergeometric function with arguments ± 1, in terms of Riemann zeta-function and Polylogarithm functions. Further we have evaluated the integrals $\int_0^1 \frac{(\ell n[1+z])^m}{z^n} \mathrm{d}z$ and $\int_a^b \frac{(\ell n[1+z])^c}{z^d} \mathrm{d}z$ with suitable convergence conditions.