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### **ABSTRACT**

Chaotic dynamics is a fascinating area of nonlinear sciences which has been extensively studied during the past few decades. Chaotic behavior is observed in different fields for instance chemical systems, electrical engineering, biological systems, secure communication and so on. In current years more and more attention has been drawn towards the control and synchronization of chaotic systems. Various kinds of synchronization phenomena have been studied, such as complete synchronization, anticipated synchronization, hybrid synchronization, projective synchronization etc. Historically, the analysis of synchronization phenomena in the evolution of dynamical systems has been a subject of active investigations since the earlier days of physics. It started in the year 1665 with the finding of Christian Huygens. He was the first to observe the anti-phase synchronization between two pendulum clocks with common frame and he noticed that the pendulum clocks swung exactly at equal frequencies and  $180^\circ$  out of phase. When he disturbed one pendulum the anti-phase state was restored within half an hour and pendulum clocks remained synchronized indefinitely, thereafter it left undisturbed. He also noticed that synchronization did not occur when the clocks were separated beyond a certain distance, or oscillated in mutually perpendicular planes. Huygens understood that the crucial interaction came from very small movements of the common frame supporting the two clocks. He also contributed a physical explanation for how the frame motion set up the anti-phase motion. Apart from this, there are several other early classic examples of synchronization of periodic systems. Generally speaking, chaos

synchronization means to control a chaotic system (called slave system) so that it follow the another chaotic system (called master system). Hyperchaos dynamics has also been comprehensively studied over the last two-three decades due to its great potential applications in many engineering oriented practical fields, such as secure communication, nonlinear circuits, lasers, control, synchronization and many more. In secure communication, the message to be transmitted is masked by a chaotic signal. We know that, the chaotic systems have one positive Lyapunov exponent. Perez and Cerderia justified that the message masked by chaotic systems are not always secure. However, Pecora established that this situation can be more useful by adapting the higher dimensional-hyperchaotic systems, having growing randomness and higher unpredictability. A hyperchaotic system in general is a chaotic system having atleast two positive Lyapunov exponents, indicating that its dynamics is spreading in various distinct directions simultaneously. It establishes that the the hyperchaotic systems possesses higher complex dynamical behavior, which is used to enhance the safety of chaotic communication systems. Therefore, the analytical design and circuitry recognition of numerous hyperchaotic signals have currently become the interesting area of research. By using various types of synchronization schemes to perform synchronization between different chaotic and hyperchaotic nonlinear dynamical systems, the computational technique is performed by using MATLAB or MATHEMATICA software in order to validate the synchronization numerically. The main aim of the computational method is to construct mathematical tools for the solution of concrete problems and development of general theory of dynamical systems. Keeping in view the above mentioned studies and their applications, we have designed two novel 4D Non linear dynamical system and some basic analysis of these novel 4D non linear systems has been done by means of dissipation, equilibrium, stability, time series, phase portrait, Lyapunov exponents, Poincare map and bifurcation diagram. Also an investigation of the multi switching and multi switching combination combination synchronization between non identical fractional order chaotic systems has been done.