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Abstract

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In the first chapter of thesis we obtain the condition which may ensure us that $\dim M \geq \dim N$, with M degenerate and N non-degenerate. We then introduce the definition of Generalized horizontally weakly conformal submersion (GHWC) between M and N . Furthermore we also obtain some results based on the curvature of GHWC submersions.

The second chapter is devoted to the lightlike (degenerate) geometry of submanifolds which was initiated by Duggal and Bejancu [3]. Later Duggal and Jin [5], Gimenez [4], B. Sahin ([11], [12]), Massamba [9] and R. Kumar et.al. [7] studied lightlike submanifolds (hypersurfaces). We introduce and study an axiom called axiom of r -submanifolds. We also improve the axiom of spheres (planes) with lightlike submanifolds given by R. Kumar, R. Rani and R. K. Nagaich in [7]. Moreover we study an application of Jacobi equation on lightlike submanifolds.

In third chapter we study some pinching relations for null sectional curvature and conclude non-existence results for CR-lightlike submanifolds of an almost Hermitian manifold. In the last of this chapter we also study some applications of the Index form and Jacobi equation [10], to conclude some more non-existence results for CR-lightlike submanifolds.

In fourth chapter first we discuss the submersion of CR- de Sitter space onto complete spacelike hypersurfaces giving the non-existence of compact spacelike hypersurfaces in CR de-Sitter space. Moreover we consider the immersion of spacelike submanifolds into de-Sitter space and prove the totally umbilicity of spacelike submanifolds under certain geometrical conditions.

In 1970 Chern, do Carmo and Kobayashi [1] generalized the result of Lawson [8] from minimal hypersurfaces to submanifolds of a sphere. Later analogously T. Ishihara [6] investigated maximal spacelike submanifolds of pseudo-Riemannian space of constant curvature (or de-Sitter space). On the other hand Deshmukh [2] studied minimal submanifolds of a sphere and concluded some interesting results and showed that minimal submanifolds of a sphere are totally geodesic under certain geometric conditions. In this chapter we generalize the results of [2] to pseudo-umbilical spacelike submanifolds of de-Sitter space.

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