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4. Title of the Thesis: A Study of Submanifolds in Almost Contact Metric Manifolds

**Abstract:** Nash's theorem enables us to consider any Riemannian manifold as a submanifold of Euclidean space. This provides us a natural motivation for the study of submanifolds of Riemannian or Semi-Riemannian manifolds. It is well known that semi-Riemannian manifold have many similarities with their Riemannian case. However, the lightlike submanifolds are different since their normal vector bundles intersect with the tangent bundle making it more interesting and difficult to study. On the other hand, the geometry of warped product plays an important role in differential geometry as well as in physics. For example, the best relativistic model of the Schwarzschild space-time that describes the outer space around a massive star or a black hole is given as warped product. The growing importance of lightlike submanifolds and warped product submanifolds in mathematical physics, in particular their extensive use in relativity and very limited information available on these submanifolds, is the motivation behind selecting the topic for the present Thesis.

The present thesis comprises five chapters. The first chapter is introductory, where we have summarized some definitions and results from submanifold theory and theory of lightlike submanifolds of a semi-Riemannian manifold which are of relevance to the subsequent chapters. It also includes the basic information on totally umbilical lightlike submanifolds. The last section of this chapter deals with the warped product submanifolds required to formulate the definition of warped product skew CR-submanifolds.

In *chapter II*, we study radical transversal and transversal lightlike submanifold of an indefinite kenmotsu manifold and give examples. We investigate integrability conditions of distributions involved in the definition of radical transversal and transversal lightlike submanifolds and obtain condition under which the induced connection is a metric connection. We also study totally contact umbilical radical transversal and transversal lightlike submanifolds

of indefinite Kenmotsu manifolds and establish the existence (or non-existence) of radical transversal lightlike submanifolds in an indefinite Kenmotsu space form. The contents of this chapter are published in **Int.J.Contemp.Math. Sciences**, 5(2010)32, 1553-1576.

The concept of hemi-slant lightlike submanifolds of an indefinite Kenmotsu manifold is given in chapter III which is a powerful structure used to study the geometry of immersed submanifolds. In section 3.2, we give an example of hemi-slant lightlike submanifold and establish two characterization theorems for the existence of such submanifolds. Continuing the study of hemi-slant lightlike submanifolds, in section 3.3, we prove that there does not exist curvature invariant hemi-slant lightlike submanifold in an indefinite Kenmotsu space form with some condition on  $c$  and obtain a geometric condition under which the induced connection on  $M$  is a metric connection. In section 3.4, we prove some theorems which ensure the existence of minimal hemi-slant lightlike submanifolds. An example of proper minimal hemi-slant lightlike submanifolds is also given. The contents of this chapter are under consideration in **ISRN Geometry**.

Chapter IV deals with the basic properties of totally contact umbilical radical screen transversal lightlike submanifolds and screen transversal anti-invariant lightlike submanifolds. In section 4.2, we obtain a geometric condition under which the induced connection  $\nabla$  on a totally contact umbilical radical screen transversal lightlike submanifold is a metric connection. The classification theorem for totally contact umbilical screen transversal anti-invariant lightlike submanifolds is given in section 4.3. The contents of this chapter are communicated in **Note di Matematica**.

In chapter V, we introduce warped product skew CR-submanifold of a cosymplectic manifold and establish a characterization theorem for warped product skew CR-submanifolds. A sharp inequality between the squared norm of the second fundamental form and the warping function for such submanifolds isometrically immersed in cosymplectic manifolds is obtained in the last section of this chapter. The equality case is also considered. The contents of this chapter are accepted for publication in **Lobachevskii Journal of Mathematics**.