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Title

"EXTENSION OF THE SITNIKOV PROBLEM TO THE RESTRICTED FOUR-BODY PROBLEM"

Abstract:

The configuration of the **Sitnikov four-body problem** consists of three equal masses (called primaries) fixed at the vertices of an equilateral triangle. The primaries are moving in circular orbits around their common centre of mass (C.M.). The fourth infinitesimal/finite mass is moving along a line perpendicular to the plane of motions of the primaries and passing through the C.M. of the primaries. To study the motions of the infinitesimal/finite mass is called Sitnikov four-body problem.

Chapter -1 is of introductory nature, we have begun our study from Chapter -2, in which primaries are taken as point masses. It has been observed that the Equation of motion for the restricted four-body problem is non-linear in nature. Due to non-linearity nature of the equation of motion, variational equations have been derived to study the linear stability of the motions of the fourth mass. A well known ***Floquet** method* has been applied for the same. Investigation has shown that motion of Sitnikov four-body problem is linearly stable for $0.7632577230 \le z_{initial} \le 2.8647296049$, where $z_{initial}$ is the value of z (position of the fourth mass) at time $t = 0$.

In chapter-3, all the three primaries are oblate spheroid contrary to the point masses taken in chapter-2. The equators of the primaries are in a same plane and coincide with the plane of motions of the three centre of mass of the primaries. A relation is established between the oblateness parameter 'A'

and the change Δl in the sides of the equilateral triangle, such that the configuration remains same throughout the motion. The same procedure of chapter-2 has been followed to investigate the linear stability of motion of the fourth mass. The range of linear stability is $0.75595489 \le z_{initial} \le 2.8574717$ for $A = 0.003.$ The above range is little wider than the range for point mass. When oblateness parameter is $A = 0.005$, the range of stability is $0.7510438 \le z_{in} \le 2.8525810$. This range is also little wider than the range for $A = 0.003.$ Sitnikov family is investigated and observed that when $z_{in} \le 2250$, Sitnikov motion exists while beyond this range Sitnikov motion terminates. Further, we have found that only **twelve critical periodic orbits/points** (points at which stability indices are ± 2) exist, from which **new three-dimensional (3-**D) **symmetric periodic orbits bifurcate.** All the twelve families of 3-D symmetric periodic orbits which bifurcate from the corresponding critical periodic orbits have been investigated. The new 3-D periodic orbits are symmetrical either with respect to the x-axis or the x z-plane or both.

In chapter-4, fourth mass is finite instead of infinitesimal. In this case the fourth mass influences the motion of the primaries. The equations of motion in the Cartesian, polar and Hamiltonian form have been derived and then system is reduced to two degrees of freedom. Maximum and minimum energy of the system are determined, which are $-\frac{9m^5}{2c^2}$ and $-\frac{9m^5}{2c^2} - \frac{9\sqrt{3}m^4 m_4}{c^2}$ respectively. Both have negative sign which indicate that motions are bounded. The permitted region of motion is very thin but increases as the value of fourth mass increases. Conditions for bounded motion and regions of possible motion in configuration space have been investigated. In the last section we have computed the Poincarè sections for the range of the energy $-0.325 \le h \le -0.297$, $m = \frac{1}{3}$, $m_4 = 10^{-2}$, $c = 0.25$. At the energy level $h = -0.318$ the chaotic region starts and at $h = -0.297$ the whole phase space filled with the chaotic orbits.