Name of the scholar: **Susheel Kumar**
Name of the supervisor: **Prof. (Mrs.) Kum Kum Dewan**
Department: **Mathematics**
Title of the thesis: **THE GROWTH OF POLYNOMIALS**

ABSTRACT

The thesis entitled, "**THE GROWTH OF POLYNOMIALS"** consists of four chapters. The first chapter deals with the inequalities for a polynomial and its derivative. In the second chapter, the inequalities concerning the maximum modulus and minimum modulus of polynomials are investigated. In the third chapter, we estimate the maximum modulus and minimum modulus for an operator B preserving inequalities between polynomials. The fourth chapter concerns with the location of the zeros of real or complex polynomials and analytic functions.

In Chapter 1, we firstly consider a class of lacunary type of polynomials having *t*-fold zeros at the origin and the remaining zeros in $|z| \geq K,$ when $K \geq 1$ and obtain a result which is an extension of a result due to Chan and Malik [*Proc. Indian Acad. Sci.*, **92** (3) (1983), 191-193]. Next, we extend a result of Dewan and Hans [*Mathematica Balkanica*, N. S. **23** (2009), 27-35] to the class of lacunary type of polynomials having *t*-fold zeros at the origin and the remaining zeros on $|z| = K,$ when $K \leq 1$. Further, in the direction of an open problem for polynomials of degree $n \geq 2$, having no zero in $|z| < K,$ when $K \leq 1$, we establish a refinement of a result due to Govil [*Proc. Nat. Acad. Sci.*, **50** (1980), 50-52] by involving some coefficients of the polynomial.

In Chapter 2, we prove a result which is a generalisation of a result due to Jain [*Indian J. Pure and Applied Math.*, **23** **(**11**)** **(**1992**),** 815-819]. The next result is concerning the minimum modulus of polynomials which is an extension of a result due to Aziz and Dawood [*J. Approx. Theory*, **54** (1988), 306-313] as well as a generalization of Jain's result [*Proc. Indian Acad. Sci. (Math. Sci.)*, **110** (2) (2000), 137-146]. Considering a class of polynomials not vanishing in $|z| < 1$, we generalize a result due to Ankeny and Rivlin [*Pacific J. Maths*., **5** (1955), 849-852] as well as a result of Jain [*Glasnik Matematicki*, **32** (52) (1997), 45-51]. Further, we consider a more general problem of investigating the dependence of

$$\left|p(Rz) - \alpha p(rz) + \beta \left\{\left(\frac{R+1}{r+1}\right)^n - |\alpha|\right\} p(rz)\right|,$$

on $\max_{|z|=1}|p(z)|$ for all α, β with $|\alpha| \leq 1$, $|\beta| \leq 1$ and $R \geq r \geq 1$ and establish a compact generalization of results proved by Bhat [Ph.D. Thesis, Vinayaka Mission University Salem, Tamilnadu, India (2009)], Aziz and Rathar [*Math. Inequal. Appl.*, **7**(3) (2004), 393-403] and Jain [*Indian J. Pure and Applied Math.*, **23** **(**11**)** **(**1992**),** 815-819]. Next, we prove a result concerning the minimum modulus of polynomials which gives a compact generalization of

results due to Bhat [Ph.D. Thesis, Vinayaka Mission University Salem, Tamilnadu, India (2009)] and Jain [*Proc. Indian Acad. Sci.* (*Math. Sci*), **110** (2) (2000), 137-146] . For the class of polynomials having no zero in $|z| < 1$, we obtain a result which is a compact generalization of results due to Bhat [Ph.D. Thesis, Vinayaka Mission University Salem, Tamilnadu, India (2009)] Aziz and Rathar [*Math. Inequal. Appl.*, **7**(3) (2004), 393-403] and Jain [*Glasnik Matematicki*,**32**(52) (1997), 45-51]

In Chapter 3, we consider an operator B which carries polynomial $p(z)$ into

$$B[p(z)] = \lambda_0 p(z) + \lambda_1 \left(\frac{nz}{2}\right)\frac{p'(z)}{1!} + \lambda_2 \left(\frac{nz}{2}\right)^2 \frac{p''(z)}{2!},$$

where λ_0, λ_1 and λ_2 are such that all the zeros of

$$u(z) = \lambda_0 + (n,1)\lambda_1 z + (n,2)\lambda_2 z^2$$

lie in the half plane

$$|z| \le \left|z - \frac{n}{2}\right|,$$

and prove some results concerning the maximum modulus and minimum modulus of $B[p(z)]$ which not only provide extensions of almost all results of chapter second to B -operator but also give compact generalizations of some well-known results due to Rahman [*Trans. Amer. Math. Soc*. **135** (1969), 295-309], Shah and Liman [*J. Inequal. Pure Appl. Math*., **9** (1) (2008), Art. 25, 1-8], Aziz and Dawood [*J. Approx. Theory*, **54** (1988), 306-313] and Lax [*Bull. Amer. Math. Soc*., **50** (1944), 509-513] and also establish some new interesting inequalities through them.

In Chapter 4, we obtain some results which improve upon the results of Dehmer [*J. Inequal. Pure Appl. Math*., **7** (1) (2006), 1-13] by obtaining smaller region in the form of an annulus containing all the zeros of a complex polynomial. Next, we extend a result due to Aziz and Zargar [*Glasnik Matematicki*, 31 (1996), 239-244] to the complex polynomials. Further, we prove a result which not only provide an extension of a result due to Aziz and Zargar [*Glasnik Matematicki*, **31** (1996), 239-244] but also improve upon a result of Aziz and Mohammad [*Cand. Math. Bull*., **27** (1984), 265-272]. Regarding the least number of zeros of a polynomial in a circle, we have been able to prove a results which provide an extension of a result due to Jain *[Proc. Indian Acad. Sci.* (*Math. Sci*.), **119** (1) (2009), 37-43]. Lastly, we also establish an extension of the above result of Jain [*Proc. Indian Acad. Sci.* (*Math. Sci*.), **119** (1) (2009), 37-43] to analytic functions with real coefficients. Besides this, thesis also contains many other results.