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**TITLE OF THE THESIS : On Extremal Properties and Zero Free Region Of Polynomials**

## ABSTRACT

The thesis entitled, "ON EXTREMAL PROPERTIES AND ZERO FREE REGION OF POLYNOMIALS" consists of six chapters. The first chapter deals with the growth of polynomials not vanishing in $|z| < k$ where $k \geq 1$, whereas in the second chapter, extremal problems related to the growth of polynomials not vanishing in $|z| < k$ where $k \leq 1$ have been investigated. The third chapter concerns with the inequalities for polynomials having all its zeros in the interior or exterior of a circle. In the fourth chapter inequalities concerning polar derivative of a polynomial have been obtained. In the fifth chapter Turàn type of inequalities have been considered for the polar derivative of a polynomial. The sixth chapter deals with the location of zeros of polynomials.

In Chapter I, we consider a class of polynomials of degree n not vanishing inside the disk $|z| \leq k$, $k \geq 1$ . For $k = 1$, we obtain results which generalize results due to Govil [Approx. Theory and its Appl., 5(1989)] as well as generalize and improve upon the results due to Dewan, Hans and Kaur [J. Interdisciplinary Math., (2010)]. We next obtain a compact generalization of results due to Govil and Nyuydinkong [J.Interdisciplinary Math., 4 (2001)] for the polynomials not vanishing inside $|z| \leq k$, $k \geq 1$ and further extend these results to lacunary type of polynomials. Lastly, in this chapter we obtain a result that depends on the location of all the zeros of $p(z)$.

In Chapter II, we study the growth of polynomials of degree n not vanishing in $|z| < k$, $k \leq 1$. First we obtain some results for the class of polynomials having all its zeros on $|z| = k$ where $k \leq 1$ and then extend them to lacunary type of polynomials. Further in this chapter, we obtain various new and interesting results for the polynomials having no zeros in $|z| < k$, $k \leq 1$ and then again extend these results to lacunary type of polynomials. Finally, we improve as well as generalize our own results proved in this chapter by restricting ourselves to the polynomial of degree $n \geq 2$.

In Chapter III, we study the growth of maximum modulus of polynomials. Firstly, we obtain few results for the class of polynomials having all its zeros in $|z| \leq k$, $k \leq 1$ where $0 \leq r \leq k \leq 1$. We then improve upon the bound of our own result by involving coefficients.

In Chapter IV, we study the extremal properties for the polar derivative of a polynomial. We firstly consider the class of polynomials having all its zeros on $|z| = k$, $k \leq 1$and extend the results due to Govil [J. Math. and Phy. Sci., 14(1980)], Dewan and Mir [Southeast Asian Bulletin of Math., 31(2007)], Dewan and Hans [Mathematica Balkanica, 23 (2009)] to the polar derivative of a polynomial and obtain a compact generalization of these results. We next, extend some well known inequalities due to Govil [Proc. Nat. Acad. Sci., 50(1980)], Dewan and Hans [Annales UMCS, Lublin, LXIII (2009)] for the class of polynomials having no zero in the disk $|z| < k$, $k \leq 1$, to the polar derivative of a polynomial. Besides these results, we have also extended some results concerning the estimate of maximum modulus of polynomial to the polar derivative for the class of polynomials having no zeros in the disk $|z| < k,\ k \leq 1$

In Chapter V, we study Turàn type inequalities for polynomials having all its zeros in $|z| \leq k$, $k \geq 1$with m-fold (where $0 \leq m \leq n$) zeros at the origin. First, we obtain a result which improves as well as generalize a result due to Aziz and Rather [Math. Inequal. Appl., 1 (1998)] and thereby obtain a compact generalization of results due to Turàn [Compositio Math.] , Govil [Proc. Amer. Math. Soc., 41(1973)], Shah [J. Ramanujan Math. Soc., 1(1996)] to the polar derivative of a polynomial of degree n with m-fold zeros at the origin. Lastly, we improve our own results proved in this chapter for polynomials of degree $n \geq 3$.

In Chapter VI, we first obtain a generalization of Eneström-Kakeya Theorem for complex coefficients which for particular cases include the results of Aziz and Mohammad [Proc. Amer. Math. Soc., 80 (1980)], Joyal, Labelle and Rahman [Canad. Math. Bull., 10 (1967)] and Bhat [Ph.D. Thesis, (2009)]. Next, we improve upon the result due to Kuniyeda, Montel and Toya [Geometry of polynomials,(1966), p. 124] and Jain [Publ. L'inst. Math., 41 (1987)] by obtaining a ring shaped region containing all the zeros of a polynomial. Besides this, some other results are also proved.