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Abstract

Generalized Fuzzy Topological Structures

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Several generalizations of open fuzzy sets are studied by several authors in fuzzy topology which are carried out by repeatedly using the interior and closure operators. These weaker forms of open fuzzy sets provide a better solution in various diverse fields of knowledge such as logic programming, pattern recognition etc. However, the study so far has been scattered and no significant effort has been taken to bring these scattered studies under one umbrella. This motivates the subject matter of this thesis entitled "**Generalized Fuzzy Topological Structures**.

The whole thesis is divided into nine chapters. In **Chapter 1**, a brief introduction, background and basic results and definitions of the research topic are given.

In **Chapter 2**, generalized open fuzzy sets or γ–open fuzzy sets, as we call them are defined using a monotonic mapping γ on a non empty set X. The study of monotonic mappings in this chapter brings the existing weaker forms of open fuzzy sets under one umbrella. It is shown that under specified conditions monotonic mappings generate γ–open fuzzy sets on a given non empty set X, which are a special class of generalized open fuzzy sets. This class of generalized open fuzzy sets is defined without using any inherent fuzzy topology on X. For example, in a fuzzy topological space, if γ is taken to be $int(cl)$, γ–open fuzzy sets coincide with pre-open fuzzy sets. Similarly, we obtain semi-open fuzzy sets for $\gamma = cl(int)$, α–open fuzzy sets for $\gamma = int(cl(int))$ and β–open fuzzy sets for $\gamma = cl(int(cl))$. Some important properties of monotonic mappings are also examined.

In **Chapter 3**, we define a generalized form of a fuzzy topological space (*GFTS*, for brevity) utilizing the two characteristics of γ–open fuzzy sets, that is, they are closed under arbitrary union and $\underline{0}$ is always γ -open. we have also introduced generalized quasi neighborhood (*g-Qnbhd*, for brevity) system of fuzzy points and came across some peculiarities in a generalized fuzzy topological space. These peculiarities are a departure from its counterpart, that is, fuzzy topological space. We have also investigated fuzzy continuity for a generalized fuzzy topological space.

In **Chapter 4**, we define interior and closure operators for generalized fuzzy topological spaces. Using interior and closure operators, generalized open fuzzy sets with respect to a generalized fuzzy topology λ on a set X are introduced. These operators can be used to identify the underlying generalized fuzzy topology, that is, taking λ as the starting point, we repeatedly apply interior and closure operators on members of λ to obtain several interesting subclasses of I^X. Each of these subfamilies contains λ and is a generalized fuzzy topology again.

The next five chapters deal with various topological properties for generalized fuzzy topological spaces. **Chapter 5**, deals with the application of fuzzy nets. In this chapter, we

introduce *g-fuzzy net*, which is a generalized form for a fuzzy net. We discuss the convergence of *g-fuzzy net* and have constructed convergence classes for generalized fuzzy topological spaces generated by *g-fuzzy net*.

Chapter 6 deals with separation axioms for generalized fuzzy topological spaces. In this chapter, we have discussed *Quasi*-T_0, T_0, T_1, T_2 axioms and regularity for generalized fuzzy topological spaces and have studied some of their properties. We have also discussed the restriction imposed on a GFTS to be a T_2 space.

Chapter 7 is devoted to the study of connectedness. In this chapter, we have introduced four connectedness notions for generalized fuzzy topological spaces, called c_i -connectedness (i = 1, 2, 3, 4). Further the notion of components is also developed.

We introduce the class of extremally disconnected generalized fuzzy topological spaces in **Chapter 8**. Apart from discussing the basic properties of extremally disconnected generalized fuzzy topological spaces, we have studied a relationship of extremal disconnectedness with fuzzy unit interval. For a generalized fuzzy topological space, a function may be both fuzzy upper semi-continuous and fuzzy lower semi-continuous and yet may fail to be fuzzy continuous. Their presence distinguishes the study of generalized fuzzy topological spaces from that of fuzzy topological spaces and general topological spaces.

In **Chapter 9** we extend the notion of N-compactness for generalized fuzzy topological spaces using g -α-net and name it as g -N-compactness. We also discuss theory of local compactness in generalized fuzzy topological spaces It is observed that g-N-compactness is hereditary with respect to generalized closed fuzzy subsets. Also, g-N-compactness and local compactness are preserved under continuity.

To sum up, this thesis has attempted to provide a framework for a generalized form of fuzzy topological spaces. It may be admitted, however, that many challenging problems still remain open in this area of research. In fact, there are many other topological structures which may be studied in the framework of generalized fuzzy topological spaces.

Our investigations have culminated into the following research papers:

1. R. D. Sarma, A. Sharfuddin and **Anubha Bhargava**, On generaliuzed open fuzzy sets, *Annals of Fuzzy Mathematics and Informatics*, Vol. 4(1) (2012), 143–154.

2. R. D. Sarma, A. Sharfuddin and **Anubha Bhargava**, Connectedness and its related notions, *International Conference on Analysis and its Applications*, November 19-21, 2011, Department of Mathematics, Aligarh Muslim University, Aligarh, India.

3. R. D. Sarma, A. Sharfuddin and **Anubha Bhargava**, Generalized open fuzzy sets in generalized fuzzy topology, *Int. J. Math. Sc.*, Vol. 9 (3-4) (2010), 343–355.

4. R. D. Sarma, A. Sharfuddin and **Anubha Bhargava**, Seperation axioms for generalized fuzzy topological spaces, *J. Comb. Inf. Syst. Sci.*, Vol. 35 (3-4) (2010), 329–340.

5. R. D. Sarma, A. Sharfuddin and **Anubha Bhargava**, Connectedness for generalized fuzzy topologies, *19th International Conference of Forum of Interdisciplinary Mathematical* (IMST 2010-FIM XIX), December 19-21, 2010, Patna University, Patna, India.

6. R. D. Sarma, A. Sharfuddin and **Anubha Bhargava**, Generalized open fuzzy sets I, *International Conference on Recent Trends in Mathematics and its Application* (ICRTMA-09), March 30-31, 2009, Department of Mathematics, Jamia Millia Islamia, New Delhi, India.

7. R. D. Sarma, A. Sharfuddin and **Anubha Bhargava**, A disconnection for generalized fuzzy topological spaces, (communicated).