TITLE OF THESIS : A STUDY OF EXTREMAL PROBLEMS, INEQUALITIES AND ZEROS OF POLYNOMIALS.

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ABSTRACT : The Thesis entitled "**A STUDY OF EXTREMAL PROBLEMS, INEQUALITIES AND ZEROS OF POLYNOMIALS** ", consists of four chapters. In Chapter I, firstly we consider the class of Lacunary type of polynomials, $p(z) = a_0 + \sum_{j=\mu}^{n} a_j z^j$, $1 \le \mu \le n$, having all its zeros in $|z| \le k$, $k \ge 1$ and obtain generalizations of some of the results due to Govil. Besides these results some other results for the class of polynomial having all its zeros on $|z| = k$, $k \le 1$ have also been obtained. Next in the same chapter, we obtain a compact generalization of a result of Lax and Aziz and Dawood, which also improve upon an inequality proved by Jain. Lastly, in this chapter we prove some inequalities for the sth derivative of a polynomial which generalizes inequalities due to Govil and Dewan and Mir.

In Chapter II, we study the growth of Maximum Modulus for Polynomials. We firstly prove a result conjectured by Aziz for the class of polynomials which does not vanish in $|z| < k$, $k < 1$ provided $|p(z)|$ and $|q(z)|$ attains their maximum at $|z| = 1$, where $q(z) = z^n p(1/z)$. Next, we consider the class of lacunary type of polynomials which does not vanish in $|z| < k$, $k \ge 1$ and present a generalization of results due to Aziz and Ankeny and Rivlin. Further, we prove some results concerning Minimum Modulus of Polynomials and thereby obtain inequalities which improve upon results of Aziz and Dawood, Jain and Aziz and Rather.

Chapter III, deals Inequalities concerning the class of polynomials P oQ(z) of degree mn such that P oQ(z) = P (Q(z)), where Q(z) polynomials of degree m and thereby obtain compact generalization of results due to Malik and Govil for the polynomials not vanishing in $|z| < k$, $k \ge 1$. We next, extend some well known inequalities for ordinary derivatives to polar derivatives due to Aziz and Shah and Aziz. Lastly, we have defined the B-operator for the composite polynomials and obtained inequalities which give compact generalization of results proved by Rahman and Shah and Liman.

In the last chapter, we study the extremal properties for the polar derivative of a polynomial. we firstly consider the class of polynomials having all its zeros in $|z| \le k$, $k \ge 1$ and extend some of our own results proved in Chapter I for ordinary derivative to polar derivative. Lastly, we consider the class of polynomials having no zeros in $|z| < k$, $k \ge 1$ and prove some inequalities for the maximum modulus of polynomials for polar derivatives which besides generalizing also improves upon results proved by Bidkham and Dewan, Dewan and Mir, Dewan and Upadhye and Aziz and Zargar.