ABSTRACT OF Ph.D. THESIS

Title of Ph.D. Thesis: *On Certain Aspects of Generalized Special Functions and Integral Operators.*
Research Scholar: *Mahendra Pal Chaudhary*
Research Supervisor: *Professor Chaudhary Wali Mohd.*

Department of Applied Sciences and Humanities, Faculty of Engineering and Technology, Jamia Millia Islamia, New Delhi-110025, India.

The present thesis comprises of **Eight Chapters**. A brief summary of the problems is presented at the beginning of each chapter and then each chapter is divided into a number of sections.

In **First Chapter** : *Brief History of Literature*, we have discussed Pochhammer's symbol and related identities; Non-terminating, terminating and truncated generlaized hypergeometric series; Wright's generalized hypergeometric function; Appell's, Kampe de Feriet's and Exton's double hypergeometric functions; Lauricella's tripple hypergeometric function; Srivastava's general triple hypergeometric function; Multivariable hypergeometric function of Srivastava-Daoust; Some series identities; Hypergeometric forms of some functions; Some summation theorems associated with well-poised series and nearly-poised series of second kind; Some transformations associated with Euler, Exton and Whipple; Normal forms; Some useful indefinite integrals; and integral operators of functions.

In **Second Chapter** : *Some Theorems Associated with Terminating and Truncated Hypergeometric Series*, we established two theorems as :

Theorem I: Suppose $\{g_n\}_{n=0}^{\infty}$ be a suitably bounded sequence of arbitrary complex numbers, defined by $2g_{n+1} = 2^{-2n}[\{c+\sqrt{(c^2-4\beta)}\}^{2n}+\{c-\sqrt{(c^2-4\beta)}\}^{2n}]$ and any values of parameters and variables leading to result which do not make sense are tacitly excluded, then $g_{n+1} = (-\beta)^n {}_2F_1[-n, n\ ;\ \frac{1}{2}\ ;\ \frac{c^2}{4\beta}]$ and $g_{n+1} + \beta\ g_n = \frac{c^{2n}}{2} + \frac{c^2(-\beta)^{n-1}(2n-1)}{2} {}_2F_1[-n+1, n\ ;\ \frac{3}{2}\ ;\ \frac{c^2}{4\beta}]_{n-2}\ ;\ n \geq 3$, where $g_0 \equiv 0$.

Theorem II: Suppose $\{h_n\}_{n=0}^{\infty}$ be a suitably bounded sequence of arbitrary complex numbers, defined by $c(2n+1)h_{n+1} = 2^{-(2n+1)}[\{c+\sqrt{(c^2-4\beta)}\}^{2n+1}+\{c-\sqrt{(c^2-4\beta)}\}^{2n+1}]$ and any values of parameters and variables leading to result which do not make sense are tacitly excluded, then $h_{n+1} = (-\beta)^n {}_2F_1[-n, n+1\ ;\ \frac{3}{2}\ ;\ \frac{c^2}{4\beta}]$ and $h_{n+1} + \beta\ h_n = \frac{c^{2n}}{1+2n} + \frac{nc^2(-\beta)^{n-1}}{3} {}_2F_1[-n+1, n+1\ ;\ \frac{5}{2}\ ;\ \frac{c^2}{4\beta}]_{n-2};\ n \geq 3$, where $h_0 \equiv 0$.

In **Third Chapter** : *Hypergeometric Solutions of Elliptic Type Single, Double Integrals of Ramanujan and others*, we established some results in continuation of earlier work of R. Y. Denis *et. al.* associated with Ramanujan's Seventh Entry of Chapter XVII of Second

Notebook. Using series iteration techniques, we obtain exact solutions of some interesting integrals given by Ramanujan, Erdélyi and Kyrala.

In **Fourth Chapter** : *Some Transformation Formulae Associated with Multiple Hypergeometric Functions*, we obtain five new transformations relating multiple hypergeometric functions given by Srivastava, Kampé de Fériet and Srivastava-Daoust, using series rearrangement technique and some summation theorems. Also, we obtain four new transformations relating double hypergeometric functions of Exton and Kampé de Fériet, by the applications of series rearrangement techniques and well known transformations of Euler and Whipple.

In **Fifth Chapter** : *Hypergeometric Transformations Motivated by the Work of Bailey, Cayley and Orr*, we obtain some hypergeometric transformations associated with double hypergeometric functions of Kampé de Fériet and Exton, using integral operational techniques.

In **Sixth Chapter** : *Multiple Series Identities Associated with Ramanujan's Integral*, we obtain analytical solutions of some unsolved incomplete elliptic type integrals of first, second and third kinds, given in Entry 7 of Chapter XVII of second notebook of Srinivasa Ramanujan. Further, we generalize these incomplete elliptic integrals in the forms of multiple series identities involving bounded multiple sequences.

In **Seventh Chapter** : *Incomplete Elliptic Integrals*, we show that incomplete elliptic integrals of first, second and third kinds, given in Entry 7 of Chapter XVII of second notebook of Srinivasa Ramanujan and several other integrals given by Ramanujan and B. C. Berndt are particular cases of our findings in Sixth Chapter.

In **Eighth Chapter** : *Hypergeometric Transformations Obtained from Seventh Entry of Chapter XVII of Second Note Book of Ramanujan and Others*, we obtain exact solutions of some unsolved incomplete elliptic integrals of first, second and third kinds, given in Entry 7 of Chapter XVII of second notebook of Srinivasa Ramanujan. We also derive hypergeometric forms of degenerated addition theorem, classical duplication formula, Jacobi's imaginary transformation, Abramowitz-Stegun imaginary transformation, Wang-Guo imaginary transformation, addition theorem, Gauss' transformation, Landen's transformation and other entries of Ramanujan involving Gauss hypergeometric function, Srivastava-Daoust double hypergeometric function, Kampé de Fériet double hypergeometric function and Srivastava's triple hypergeometric function.

A detailed bibliography appears at the end, with the author names in alphabetical order. References to the bibliography are numbered. The thesis includes reprints of few published research papers.