Abstract of Ph.D. Thesis

<u>**Title of Ph.D. Thesis**</u>: Elucidation of Theoretical and Analytic Aspects of Multivariable Hypergeometric Functions

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Multiple hypergeometric functions constitute a natural generalization of the Gaussian hypergeometric functions of one variable. Since the introduction of double hypergeometric functions by Appell and triple hypergeometric functions by Lauricella, numerous papers by many workers have been published and the theory has been considerably extended. An extensive study has been made in Europe, America and India of these multiple functions, which has produced almost an explosion of knowledge on the subject.

The purpose of the present research is to study generalized functions, which can provide a unification scheme for the well known special functions (scattered in the existing literature). Especially the generalization of the special functions which are widely used in various areas of applied mathematics have been dealt with. The results thus established do not merely generalize the results given earlier by various workers but also yield a number of new results. The findings can be applied to physics, statistics, engineering and other branches of science.

The present thesis comprises of SEVEN CHAPTERS. A brief summary of the problems is presented at the beginning of each chapter and then each chapter is divided into a number of sections.

The aim of the CHAPTER 1, is to introduce several classes of special functions, which occur rather frequently in the subsequent chapters.

In CHAPTER 2, we obtain the generalizations and unifications of identities associated with hypergeometric function of one variable ${}_AF_B$ and Appell's hypergeometric function of two variables F_1,F_2,F_3,F_4 due to MacRobert and Sharma respectively, in the form of identities associated with multiple Gaussian hypergeometric functions H, G of Exton and $F^{(3)}$ of Srivastava.

In CHAPTER 3, we obtain some new transformations relating quadruple hypergeometric function $F^{(4)}$ of Srivastava and quadruple hypergeometric functions D_5, K_{12}, K_{13} of Exton. Two correct forms of an erroneous transformation of Exton are also given.

The main object of CHAPTER 4 is to obtain hypergeometric reduction formulas involving Appell's double hypergeometric function of fourth kind F_4 and Srivastava-Daoust hypergeometric function of three variables, by means of integral operational technique. Watson's summation theorem for Clausenian function ${}_3F_2$ having unit

argument is obtained as special case. An erroneous hypergeometric reduction formula of Harold Exton for Gauss function ${}_2F_1$ is also corrected here.

The main object of CHAPTER 5 is to determine the correct form of a summation theorem for ${}_2F_1[a, b; \frac{(a+b-1)}{2}; \frac{1}{2}]$, given in the monograph of Prudnikov *et. al.* Explicit form of some contiguous and associated summation theorems in the form of ${}_2F_1[a, b; \frac{(a+b+N)}{2}; \frac{1}{2}]$ and ${}_2F_1[a, N-a;\ c\ ; \frac{1}{2}]$ are also obtained for $N = -5, -4, -3, -2, -1, 0, 2, 3, 4, 5$.

Similarly in a series of papers generalized Kummer's summation theorem, generalized Bailey's summation theorem, generalized Watson's summation theorem, generalized Whipple's summation theorem and generalized Dixon's summation theorem, are given whose numerator and denominator parameters are arranged in tabular form.

In CHAPTER 6, we obtain interesting finite combinations of Srivastava's general triple hypergeometric function $F^{(3)}$ as a bilateral generating function for Gauss's ordinary hypergeometric function of one variable ${}_2F_1$ and Exton's double hypergeometric polynomials X, by series rearrangement technique.

In CHAPTER 7 we obtain five general double series and Gaussian hypergeometric reduction formulae for Srivastava-Daoust double hypergeometric functions, using series rearrangement techniques.

The present chapter contains five general double series identities which extend and generalize the theorems. These theorems given in section 7.2 provide connections with various classes of well known hypergeometric functions. Some applications of these theorems are given in section 7.3.

A detailed bibliography appears at the end; with the author names in alphabetical order. References to the bibliography are numbered. The thesis includes appendices which contain reprints of a few published papers and Gamma tables, etc.