Contribution to the study of Multiple Gaussian Hypergeometric Functions

Author

Mohd. Sadiq Khan

Supervisor

Dr. Mohd. Idris Qureshi
Department of Applied Science & Humanities
Jamia Millia Islamia

Co-Supervisor

Prof. M. A. Pathan
Department of Mathematics, AMU
Aligarh

The entire work is mathematically sound and provide insight into the structure of special functions in general and hypergeometric functions in particular. It is certainly a valuable contribution in the further development of the subject. The thesis comprises of seven chapters.

Chapter 1 contains the definitions and notations of several special functions with their convergence conditions. These special functions rather occur frequently in the study of summation, transformation and generating functions. This chapter is intended to provide an introduction to variety of hypergeometric functions together with other special functions used in subsequent chapters of the thesis.

In Chapter 2, a new transformation formula for Exton's double hypergeometric function is derived by using the technique of the manipulation of series together with the well-known Watson sunimation theorem. Several transformation formulae involving ${}_4F_3$ with unit argument and Srivastava-Daoust functions of two variables are deduced from the main result. It is also shown how the main transformation formula can be applied to derive well-known Gauss's quadratic transformation.

In Chapter 3, the Saalschutz and Gauss summation theorems are utilized to derive two new reduction formulae involving Exton's double hypergeometric functions. Some well-known quadratic transformations for ${}_2F_1$, Whipple's transformation and Watson's summation theorem for ${}_3F_2$ are obtained as special cases of main results.

The Chapter 4 deals with the multiplication formulas involving Hypergeometric polynomials ${}_2F_1$, generalized hypergeome,tric polynomials ${}_{s+k+l}F_{s+k+1}$, Lauricelia's triple hypergeometric functions F_4, F_8, $F_A^{(3)}$. Srivastava's triple hypergeometric functions H_A and Exton's quadruple hypergeornetric functions K_{10}, K_{11} and K_{13}. These formulas are proved by using known Eulerian integral representations of multiple hypergeometric functions. Some special cases are also discussed as the application of

the main results.

Chapter 5 is devoted to the investigation of general multiple serie identities which extend and generalize the results of Pathan, Qureshi and Khan, Bailey, Pathan and Preece. These series identities will be seen to be extremely useful, in that most properties of hypergeometric series carry over naturally and simply for these identities and provide connections with various classes of well-known hypergeometric functions and even new representations for special cases of these functions. Some generalizations of Preecean theorem involving the product of two Kummer's functions ${}_1F_1$ are obtained by using Dixon's theorem and Srivastava's identities. Its special cases yield various new transformations and reduction formulae involving Pathan's quadruple hypergeometric function $F_a^{(4)}$, Srivastava's quadruple hypergeometric function $F^{(4)}$ and triple hypergeometric function $F^{(3)}$, Lauricelia's quadruple hypergeometric function $F_A^{(4)}$and triple hypergeometric function $F_A^{(3)}$, Exton's multiple hypergeometric functions K_{10}, K_{13}, X_8, ${}^{(k)}H_2^{(n)}$ Erdelyi multiple hypergeometric function $H_{n,k}$, triple hypergeometric function of Khan and Pathan, Kampe de Feriet's double hypergeometric function, Appell's double hypergeometric function of second kind F_2 and Srivastava-Daoust function.

Chapter 6 is devoted to the investigation of general multiple series identities which extend and generalize theorems of Bailey. These theorems will be seen to be extremely useful, in that most properties of hypergeonietric series carry over naturally and simply for these identities and provide connections with various classes of well-known hypergeometric functions and even new representations for special cases of these functions. Some generalizations of Bailey's theorem involving the product of two ${}_2F_0$, are obtained by using Whipple's summation theorem and Srivastava's identities. Its special cases yield various new transformations and reduction formulae involving Pathan's quadruple hypergeometric function $F_p^{(4)}$, Srivastava's quadruple hypergeometric function $F^{(4)}$ and triple hypergeometric function $F^{(3)}$, Exton's multiple hypergeometric functions, Erdelyi's multiple hypergeometric function $H_{n,k}$, Lauricella's triple hypergeometric function $F_B^{(3)}$, Kampe de Feriet's double hypergeometric function and Appell's double hypergeometric function of third kind F_3. Some known and unknown results are also obtained in this chapter.

In the *last chapter* of the thesis, eight new general theorems on generating relations for a certain sequence of functions using series rearrangement techniques and closed hypergeometric reduction formulas. Theses theorems are then applied to derive a

number of new and known generating relations due to Shively, Toscano, Rainville, Burchnall and Krall and Frink.

These research works has been published, accepted and communicated for publication in different International, National and proceeding of the conferences.

I hope that the investigations carried over in the persent thesis on multiple hypergeoinetric functions are of some interest and will provide insight into the structure of special function in general and hypergeometric functions in special and would raise some problems for further research. It has been our and to show that there is a truthful interaction and connection between the hypergeometric functions of different types and nature, while at the same time demonstrating that multiple hypergeometric possesses a special character and have particular applications, which clearly make them not just a subtopic generalizations in the study of hypergeometric functions.